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Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD

CLASS - 8

Question Paper Code : 4P104

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A	A	C	B	A	A	A	A	A	D

SOLUTIONS

MATHEMATICS - 1 (MCQ)

01. (C) $\sqrt{1^3+2^3+3^3+....+9^3} = \sqrt{2025} = 45$

02. (D) $\sqrt[3]{100^3-3\times100^2\times97+3\times100\times97^2-97^3}$

$= \sqrt[3]{(100-97)^3} = 3$

03. (A) Given $P\left(1+\frac{R}{100}\right)^2 - P - \frac{PTR}{100} = \text{Rs. } 1.5$

$P\left(1+\frac{5}{100}\right)^2 - P = \frac{P\times2\times5}{100} = \text{Rs. } 1.5$

$\frac{441P}{400} - P - \frac{P}{10} = \text{Rs. } 1.5$

$\frac{441P-400P-40P}{400} = \text{Rs. } 1.5$

$\frac{P}{400} = \text{Rs. } 1.5 \Rightarrow P = \text{Rs. } 600$

04. (D) Originally, let there be x men. Less men $\Rightarrow$ more days.

$\therefore \quad (x-10) : x :: 100 : 110$

$\Rightarrow (x-10)\times110 = x\times100$

$\Rightarrow 10x = 1100 \Rightarrow x = 110$

05. (B) Let the cost of pressure cooker be Rs x. & sales be y.

$\therefore$ Total amount = Rs xy.

Given new cost = 80% $x = \frac{80}{100}x = \frac{4x}{5}$

New sales = 180% $y = \frac{180}{100}y = \frac{9}{5}y$

New amount = $\frac{4x}{5} \times \frac{94}{5} = \frac{36xy}{25}$

Increased sales = $\frac{36xy}{25} - xy = \frac{11xy}{25}$

Increased sales %

$= \frac{(11xy)}{\frac{25}{xy}} \times 100 = \frac{(11)}{25} \times 100$

= 44 %

06. (C) Let x be a number. Then the other two numbers are $2x$ and $3x$. According to the problem,

$x + 2x + 3x = 12$

$\Rightarrow 6x = 12 \Rightarrow x = 2$

$\therefore$ The largest of the numbers is $3x = 6$

07. (C) $3^n = 729 \Rightarrow 3^n = 3^6 \Rightarrow n = 6$

$\therefore\ 3^{3n+1} = 3^{3(6)+1} = 3^{18+1} = 3^{19}$

08. (B) Given $6x^2 = 1536\ cm^2$

$a^2 = \frac{1536}{6} cm^2 = 256\ cm^2$

$a^2 = (16cm)^2$

a = 16 cm

Volume = $a^3 = (16cm)^3 = 4096\ cm^3$

09. (C) $(x^2 - 1)(x^4 + x^2 + 1) = (x^2 - 1)[(x^2)^2 + x^2(1) + 1^2]$

$= (x^2)^3 - 1^3 = x^6 - 1$

10. (C) Side of square $= \sqrt{(4a^2 + 12ab + 9b^2)}cm$

$= \sqrt{(2a+3b)^2}cm$ = (2a + 3b) cm

Perimeter of a square = 4s = 4(2a + 3b) cm

= (8a + 12b) cm

11. (C) Let $x = 0.\overline{621} = 0.621\,621\,621....$

$\therefore$ 1000x = 1000 × 0.621 621 621....

$1000x = 621.621\,621\,621....$

$x = \quad 0.621\,621\,621....$

$\therefore\ 999x = 621$

$x = \frac{\cancel{621}^{69^{23}}}{\cancel{999}_{111_{37}}} = \frac{23}{37}$

$\therefore\ 0.\overline{621} = \frac{23}{37}$

12. (B) If x = a + b + c then

$\frac{a+b-(a+b+c)}{c} + \frac{a+c-(a+b+c)}{b}$

$+ \frac{c+b-(a+b+c)}{a} + \frac{4(a+b+c)}{(a+b+c)} = 1$

$\Rightarrow \frac{a+b-a-b-c}{c} + \frac{a+c-a-b-c}{b}$

$+ \frac{c+b-a-b-c}{a} + 4 = 1$

$\Rightarrow -\frac{c}{c} + \frac{(-b)}{b} + \frac{(-a)}{a} + 4 = 1$

$\Rightarrow -1 - 1 - 1 + 4 = 1$

$\Rightarrow 1 = 1$

Hence x = (a + b + c)

13. (A) $(1^3 + 2^3 + 3^3 + ... + 73)^{-3/2} = (1 + 8 + 27 + 64 + 125 + 216 + 343)^{-3/2}$

$= (784)^{-3/2}$

$= 28^{2\times-\frac{3}{2}}$

$= 28^{-3}$

$= \frac{1}{(28^3)}$

$= \frac{1}{21952}$

14. (D)

$$\left(\frac{3}{5}x^2y-\frac{7}{3}xy^2\right)\left(\frac{3}{7}xy^2+\frac{5}{3}x^2y\right)=\frac{3}{5}x^2y\left(\frac{3}{7}xy^2+\frac{5}{3}x^2y\right)-\frac{7}{3}\times y^2\left(\frac{3}{7}xy^2+\frac{5}{3}x^2y\right)$$

$$=\frac{3}{5}\times\frac{3}{7}x^3y^3+\frac{3}{5}\times\frac{5}{3}x^4y^2-\frac{7}{3}\times\frac{3}{7}x^2y^4-\frac{7}{8}\times\frac{5}{3}x^3y^3$$

$$=\frac{9}{35}x^3y^3+x^4y^2-x^2y^4-\frac{35}{9}x^3y^3$$

$$=\frac{9}{35}x^3y^3-\frac{35}{9}x^3y^3+x^4y^2-x^2y^4$$

$$\frac{81x^3y^3-1225x^3y^3}{315}+x^4y^2-x^2y^4$$

$$-\frac{1144x^3y^3}{315}+x^4y^2-x^2y^4$$

$$=x^4y^2-\frac{1144x^3y^3}{315}-x^2y^4$$

15. (C) P = Rs. x, A = 27

r = 20% p.a.

n = 3 years.

$$\therefore y=x\left(1+\frac{20}{100}\right)^3\Rightarrow y=x\left(\frac{6}{5}\right)^3 s$$

$$\Rightarrow\frac{y}{x}=\frac{216}{125}$$

16. (B) Given $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\frac{a+b}{2}$

$2(a^{n+1}+b^{n+1})=(a+b)(a^n+b^n)$

$2a^{n+1}+2b^{n+1}=a\times a^n+ab^n+ba^n+b\times b^n$

$\cancel{a^{n+1}}+a^{n+1}+b^{n+1}+\cancel{b^{n+1}}=\cancel{a^{n+1}}+ab^n+a^nb+\cancel{b^{n+1}}$

$\therefore\quad a^{n+1}-a^nb=ab^n-b^{n+1}$

$a^n(a-b)=b^n(a-b)$

$a^n=b^n$

$\therefore\quad n=0$ [$\because$ a & b are coprimes]

(OR) Verify from options

17. (C) LHS $=x^{\frac{a+b-c}{(a-c)(b-c)}}\cdot x^{\frac{b+c-a}{(b-a)(c-a)}}\cdot x^{\frac{a+c-b}{(a-b)(c-b)}}$

$$=x^{\frac{(c-a-b)}{(c-a)(b-c)}}\cdot x^{\frac{(a-b-c)}{(c-a)(a-b)}}\cdot x^{\frac{b-c-a}{(b-c)(a-b)}}$$

$$=x^{\frac{(c-a-b)}{(c-a)(b-c)}+\frac{(a-b-c)}{(c-a)(a-b)}+\frac{b-c-a}{(b-c)(a-b)}}$$

$$=x^{\frac{(a-b)(c-a-b)+(b-c)(a-b-c)+(c-a)(b-c-a)}{(a-b)(b-c)(c-a)}}$$

$$=x^{\frac{ac-a^2-ab-bc+ab+b^2+ab-b^2-bc-ac+bc+c^2}{(a-b)(b-c)(c-a)}}$$

$+bc-c^2-ac-ab+ac+a^2$

$$=x^{\frac{0}{(a-b)(b-c)(c-a)}}=x^\circ=1$$

18. (B) Let breadth be 'x' cm

Given $l=3x$

Given $(3x-3)(x+5)-(3x)(x)=57$

$3x^2+15x-3x-15-3x^2=57$

$12x=57+15=72$

$x=6$

$\therefore\ l=3x=18$

P = $2(l+b)$ = 2 (24 cm) = 48 cm.

19. (C) SP after 12% discount = MP $\frac{(100-12)}{100}$

$$=MP\times\frac{88}{100}$$

SP after 20%

on above = MP $\frac{88}{100}\times\frac{(100-20)}{100}$

$$=MP\times\frac{88}{100}\times\frac{80}{100}=MP\times\frac{88}{125}$$

Discount

percentage = $\frac{MP-MP\left(\frac{88}{125}\right)}{MP}\times100$

$$=\frac{(125-88)}{125\times MP}\times100$$

$$=\frac{37}{5}\times4=\frac{148}{5}=29\frac{3}{5}\%$$

(OR)

$$100-\frac{(100-d_1)(100-d_2)}{100^{n-1}}$$

$$=100-\frac{88\times80}{100}=\frac{500-352}{5}=\frac{148}{5}$$

(OR) $a+b-\frac{ab}{100}$

20. (B) Circumfurence of circle = Permeter of a rectangle

$2\pi r = 2(18.7 + 14.3)$

$\frac{22}{7} \times r = 33$

$r = \cancel{33}^{3} \times \frac{7}{\cancel{22}_{2}} = \frac{21}{2} = 10.5$ cm

21. (A) Let a = 20252025, b = 20242025 then

LHS = ab – (b – 1) (a + 1)

= ab – (ab + b – a – 1)

= $\cancel{ab} - \cancel{ab} - b + a + 1$

= a – b + 1

= 20252025 – 20242025 + 1

= 10000 + 1

= 10001

22. (A) Given $\angle A + \angle C + \angle B + \angle C = 120° + 140°$

$\therefore \angle A + \angle B + 2\angle C = 260°$

$100° + 2\angle C = 260°$ [$\because \angle A + \angle B = 100°$]

$2\angle C = 260° - 100° = 160°$

$\angle C = \frac{160°}{2} = 80°$

But given $\therefore \angle A + \angle C = 120°$

$\angle A + 80° = 120°$

$\angle A = 120° - 80° = 40°$

23. (D) $\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\right)\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}\right)$

$= a^{\frac{1}{3}}\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}\right) + b^{\frac{1}{3}}\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}\right)$

$= \left(\begin{array}{l} a^{\frac{1}{3}} \times a^{\frac{2}{3}} - a^{\frac{1}{3}} \times a^{\frac{1}{3}} \times b^{\frac{1}{3}} + a^{\frac{1}{3}}b^{\frac{2}{3}} \\ \quad + a^{\frac{2}{3}}b^{\frac{1}{3}} - a^{\frac{1}{3}} \times b^{\frac{1}{3}} \times b^{\frac{1}{3}} + b^{\frac{1}{3}} + b^{\frac{1}{3}} \end{array}\right)$

$= \left(a^{\frac{1}{3}+\frac{2}{3}} - a^{\frac{2}{3}}b^{\frac{1}{3}} + a^{\frac{1}{3}}b^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{1}{3}+\frac{2}{3}}\right)$

$= a^{\frac{1+2}{3}} + b^{\frac{1+2}{3}} = a^{\frac{3}{3}} + b^{\frac{3}{3}} = (a + b)$

24. (D) $\sqrt{(1234567)^2 - 2469133}$

$= \sqrt{(1234567)^2 - 2469133 - 1 + 1}$

$= \sqrt{(1234567)^2 - 2469134 + 1}$

$= \sqrt{(1234567)^2 - 2(1234567)(1) + 1^2}$

$= \sqrt{(1234567 - 1)^2}$

= 1234566

25. (C) $\alpha\beta = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$

$= \frac{(-b)^2 - \left(\sqrt{b^2 - 4ac}\right)^2}{4a^2}$

$= \frac{b^2 - \left(b^2 - 4ac\right)}{4a^2} = \frac{b^2 - b^2 + 4ac}{4a^2}$

$= \frac{4ac}{4a^2} = \frac{c}{a}$

26. (B) Volume of each cube

= $(7 \text{ cm})^3 = 343 \text{ cm}^3$

$\therefore$ Volume of 10 cubes = 3, 430 cm^3

27. (A) Let the initial length and the breadth of the rectangle be *x* and *y* respectively.

$\therefore$ Initial area = *xy*

Now, new length = $x + x \times \frac{50}{100} = \frac{3x}{2}$

And new breadth = $y - y \times \frac{25}{100} = \frac{3y}{4}$

$\therefore$ New area = $\frac{3x}{2} \times \frac{3y}{4} = \frac{9xy}{8}$

Clearly, new area is more than the initial area.

$\therefore$ Increment in percent in area

$= \frac{\frac{9xy}{8} - xy}{xy} \times 100\% = 12.5\%$

28. (D) If the length of the diagonals are not equal, then the quadrilateral can be a rhombus but not a square.

29. (B)

4 seat table	5 seat table	Total tables	Total people	
9	10	19	9 x 4 + 10 x 5 = 86	✗
10	9	19	10 x 4 + 9 x 5 = 85	✗
11	8	19	11 x 4 + 8 x 5 = 84	✓

Hence, 8 tables can seat 5 people.

30. (B) $\frac{8^a}{2^a} = 2$, since $b \neq 0$

$\therefore 2^{3a-a} = 2^1$

$\Rightarrow 2a = 1$

$\therefore a = \frac{1}{2}$

MATHEMATICS - 2 (MAQ)

31. (A,B,C,D) All options are factors of the given number.

32. (A,B,C)

33. (B,C,D)

Given, $3\angle A = 4\angle B = 4\angle C = 6\angle D$ ..... (i)

Let (i) = k

$\therefore$ we get, $\frac{k}{3} + \frac{k}{4} + \frac{k}{4} + \frac{k}{6} = 360^\circ$

[sum of all angles of a quadrilateral = 360^o]

$\Rightarrow \frac{4k+3k+3k+2k}{12} = 360^o$

$\Rightarrow$ $k = 360^o$

$\Rightarrow$ $k = 360^o$

So, the angles are 120^o, 90^o, 90^o, 60^o.

34. (A,C) $x^4 + 2x^2 + 9 = (x^2)^2 + 3^2 + 2x^2$

$= (x^2)^2 + 3^2 + 2 \times x^2 \times 3 - 2x^2 \times 3 + 2x^2$

$= (x^2 + 3)^2 - 6x^2 + 2x^2$

$= (x^2 + 3)^2 - 4x^2$

$= (x^2 + 3)^2 - (2x)^2$

$= (x^2 + 2x + 3)(x^2 - 2x + 3)$

35. (A,B,D)

Given

$x_1y_1 = x_2y_2 = x_3y_3 = x_4y_4 = x_5y_5$

$12a = 6 \times 16 = 8b = 0.5c = 4d.$

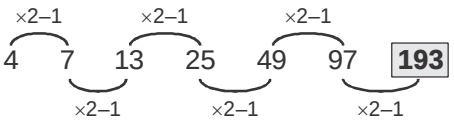
$\therefore$ $12a = 16 \times 6 \Rightarrow a = 8$

$8b = 6 \times 16 \Rightarrow b = 12$

$0.5c = 6 \times 16 \Rightarrow c = 192$

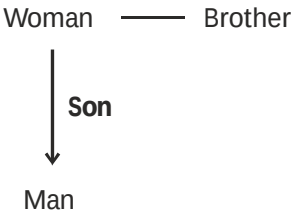
$4d = 6 \times 16 \Rightarrow d = 24$

REASONING

36. (D) 

37. (C) $(16 \times 2) + 8 = 40$; $(11 \times 2) + 5 = 27$;

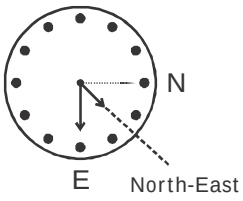
$(4 \times 2) + 11 =$ **19**

38. (B) 

39. (C) The words given in the box are related to crop production. So, the missing word is SOW.

40. (B) Small dark fill circle is not towards the vertex, as in the other figures.

41. (A) 

42. (A) 

The hour hand points to the North-East direction.

43. (C) $\sqrt{81} + \sqrt{64} + \sqrt{144} + \sqrt{36} = 9 + 8 + 12 + 6$

$= 35$

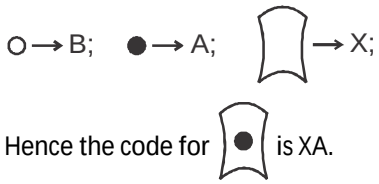
$\sqrt{49} + \sqrt{25} + \sqrt{121} + \sqrt{16} = 7 + 5 + 11 + 4$

$= 27$

$\sqrt{169} + \sqrt{625} + \sqrt{64} + \sqrt{81}$

$= 13 + 25 + 8 + 9 = 55$

44. (B) A copy of the first figure flips, grey fill turns black and together form a complete circle in the first pair. Similarly in the second pair a triangle is formed.

45. (A) 

CRITICAL THINKING

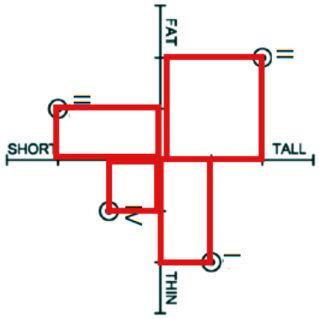
46. (A) T had Red (R) and Green (G)

J had Yellow (Y) and Red (R)

S had Green (G) and Blue (B)

D had Blue (B) and Yellow (Y)

Friend	Crayon 1	Crayon 2
T	R	G
J	Y	R
S	H	B
D	B	Y

47. (A) 

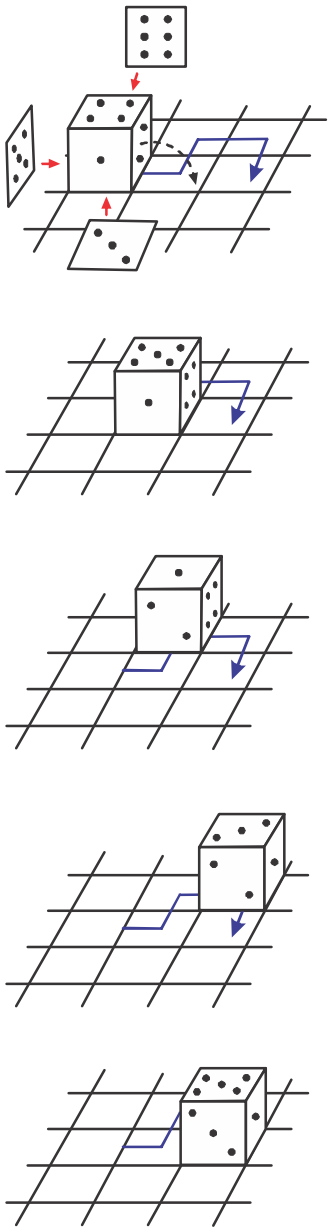
48. (A) The assertion (A) states that jellyfish use their tentacles to subdue prey. This is a true statement.

The reason (R) states that jellyfish have stinging cells (called cnidocytes) in their tentacles. This is also a true statement.

The stinging cells in the tentacles are indeed used to inject toxins into prey, which helps to subdue or paralyze them.

Therefore, both (A) and (R) are true, and (R) is the correct explanation of (A).

Answer : (A) If both (A) and (R) are true and (R) is the correct explanation of (A).

49. (A) 

50. (D) The correct answer to this question would have been 25 kg. In order for the seesaw to be in equilibrium, the net force must equal zero. Moment of a force = force x distance to pivot. Therefore, the moment of a force on the left side of the seesaw is 10 kg x 5 = 50. The right side must equal 50 for the seesaw to be balanced, therefore 50 divided by 2 (2 meters distance from pivot) = 25.